

Robust Optical Flow Estimation of Double-Layer Images under Transparency or Reflection (Supplementary Material)

1. Details of Energy Minimization

More details of our minimization techniques to solve the latent image layers and the optical flows are presented as follows.

1.1. Update the latent layers

As shown in the main paper, given the current optical flow estimates U and V , the latent image layers L_2, L'_2 can be updated by solving the following optimization problem:

$$\begin{aligned} \min_{L_2, L'_2} & \|(\mathbf{I} - L_2)(\mathbf{X}) - (\mathbf{I}' - L'_2)(\mathbf{X} + \mathbf{U})\| + \|L_2(\mathbf{X}) - L'_2(\mathbf{X} + \mathbf{V})\| \\ & + \lambda_L (\|\nabla_x(\mathbf{I} - L_2)\| + \|\nabla_x(\mathbf{I}' - L'_2)\| + \|\nabla_x L_2\| + \|\nabla_x L'_2\|) \end{aligned} \quad (1)$$

subject to $0 \leq L_2 \leq \min(\mathbf{I}, c)$, $0 \leq L'_2 \leq \min(\mathbf{I}', c)$.

This is a convex optimization problem defined on L_2 and L'_2 , and the cost function can be arranged into:

$$\begin{aligned} \min_{\mathbf{l}} & \|\mathbf{A} \cdot \mathbf{l} - \mathbf{b}\|, \\ \text{subject to} & lb_i \leq l_i \leq ub_i, \end{aligned} \quad (2)$$

where \mathbf{A} and \mathbf{b} encode all the ℓ_1 constraints on latent layers, \mathbf{l} is a column vector containing elements in L_2 and L'_2 , and lb_i and ub_i are constant bounds.

For efficient minimization, we propose to solve the problem by using iteratively reweighted least squares (IRLS).

There are some issues need to be considered in using IRLS to solve our problem. As shown in Eq. (1) and Eq. (2), the solution vector is confined by both lower and upper bounds. To deal with these bounds, one may add a projection operator inside the iteration loop of IRLS to guarantee the solution bounded (see Line 6 in Fig. 1). However, it can be seen from Eq. (1) that, when the bounds are ignored, there is a *offset scale ambiguity*: adding any constant scalar to L_2, L'_2 does not affect the objective function. To resolve this ambiguity, we shift the solution vector such that it minimizes the objective function after projection. The shifting scalar can be efficiently computed via a 1D search, as shown in Line 5 of Fig. 1.

We found our modified IRLS algorithm outlined in Fig. 1 to be both effective and efficient in solving the large-scale sparse linear problem.

1.2. Update the two flow fields U and V

Given current layer estimates L_2, L'_2 , and $L_1 = \mathbf{I} - L_2$, $L'_1 = \mathbf{I}' - L'_2$, the next step is to update the associated two flow fields U and V . This is done by solving the following optimization problem:

$$\begin{aligned} \min_{U, V} & \|L_1(\mathbf{X}) - L'_1(\mathbf{X} + \mathbf{U})\| + \|L_2(\mathbf{X}) - L'_2(\mathbf{X} + \mathbf{V})\| \\ & + \lambda_F (\|U\|_{\text{TGV}^k} + \|V\|_{\text{TGV}^k}). \end{aligned} \quad (3)$$

which can be divided in to the following two problems as two flow fields are independent with each other:

$$\min_U \|L_1(\mathbf{X}) - L'_1(\mathbf{X} + \mathbf{U})\| + \lambda_F \|U\|_{\text{TGV}^k}, \quad (4)$$

$$\min_V \|L_2(\mathbf{X}) - L'_2(\mathbf{X} + \mathbf{V})\| + \lambda_F \|V\|_{\text{TGV}^k}. \quad (5)$$

We now detail our optimization techniques for the above optical flow problems.

To solve Eq. (4) and (5), we use quadratic relaxation and introduce an auxiliary flow field to decouple the BCC term and regularization term, similar to [5, 4]. Taking the minimization of U in Eq. (4) for example, we introduce an auxiliary flow field Λ , and relax Eq. (4) as

$$\begin{aligned} \min_{U, \Lambda} & \|L_1(\mathbf{X}) - L'_1(\mathbf{X} + \mathbf{U})\| + \\ & \sum_{i=1,2} \frac{1}{2\theta} (U_i - \Lambda_i)^2 + \lambda_F \sum_{i=1,2} \|\Lambda_i\|_{\text{TGV}^k}, \end{aligned} \quad (6)$$

where θ is a small constant (0.2 in our implementation) such that Λ is U 's close approximation, U_1, Λ_1 are horizontal flows and U_2, Λ_2 are vertical flows. Eq-(6) is minimized via alternately optimizing U and Λ . When solving for U , we use the first order Taylor approximation to linearize $L'_1(\mathbf{X} + \mathbf{U})$ as in [2, 5], and obtain a closed-form solution for U ; coarse-to-fine pyramid is used to ensure accurate linearization. When solving for Λ_i , we opt for the recent first-order primal-dual technique [1] to solve the problems of $\text{TV-}\ell_2$ (*i.e.* $k = 1$) and $\text{TGV}^2\text{-}\ell_2$ (*i.e.* $k = 2$). See Fig. 2 and Fig. 3 for the algorithms we applied for $\text{TV-}\ell_2$ and $\text{TGV}^2\text{-}\ell_2$, respectively.

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1: Input  $\mathbf{l}^{(0)}$ .
2: for  $t = 1, \dots, n$  do
3:    $\mathbf{w}^{(t)} = [\dots, w_j^{(t)}, \dots]$ , where  $w_j^{(t)} = |\mathbf{A}_j \cdot \mathbf{l}^{(t-1)} - b_j|^{-1}$  ▷ Reweighting
4:    $\mathbf{l}^{(t)} = (\mathbf{A}^T \mathbf{W}^{(t)} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W}^{(t)} \mathbf{b}$ , where  $\mathbf{W}^{(t)} = \text{diag}(\mathbf{w}^{(t)})$  ▷ Least square solver
5:    $s = \text{argmin}_s \sum_j |\mathbf{A}_j \cdot \text{clip}_{[\mathbf{lb}, \mathbf{ub}]}(\mathbf{l}^{(t)} + s \cdot \mathbf{1}) - b_j|$  ▷ Shift
     where  $\text{clip}_{[\mathbf{lb}, \mathbf{ub}]}(\mathbf{l})$  enforces  $l_i \leftarrow \max(\min(l_i, \text{ub}_i), \text{lb}_i), \forall i$ 
6:    $\mathbf{l}^{(t)} = \text{clip}_{[\mathbf{lb}, \mathbf{ub}]}(\mathbf{l}^{(t)} + s \cdot \mathbf{1})$  ▷ Projection
7: end for

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Figure 1: Iteratively reweighted least squares (IRLS) with shift-projection operation in updating latent layers

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1: Set  $\mathbf{X}^{(0)} = \bar{\mathbf{X}}^{(0)} = \mathbf{F}$ ,  $\mathbf{P}^{(0)} = \mathbf{0}$ ,  $\sigma = \tau = \frac{1}{\sqrt{8}}$ 
2: for  $t = 1, \dots, n$  do
3:    $\mathbf{P}^{(t)} = \mathcal{P}_{\|\cdot\|_2 \leq 1}(\mathbf{P}^{(t-1)} + \sigma \nabla \bar{\mathbf{X}}^{(t-1)})$ , where  $(\mathcal{P}_{\|\cdot\|_2 \leq 1}(\hat{\mathbf{P}}))_{i,j} = \hat{\mathbf{P}}_{i,j} / \max(1, \|\hat{\mathbf{P}}_{i,j}\|_2)$  ▷ Dual update
4:    $\mathbf{X}^{(t)} = \frac{\theta \bar{\mathbf{X}}^{(t-1)} + \theta \tau \text{div}(\mathbf{P}^{(t)}) + \tau \mathbf{F}}{\theta + \tau}$  ▷ Primal update
5:    $\bar{\mathbf{X}}^{(t)} = 2\mathbf{X}^{(t)} - \bar{\mathbf{X}}^{(t-1)}$ 
6: end for

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Figure 2: The primal-dual algorithm to solve $\text{argmin}_{\mathbf{X}} \frac{1}{2\theta} \|\mathbf{X} - \mathbf{F}\|_2^2 + \|\mathbf{X}\|_{\text{TV}}$ in updating flow fields

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1: Set  $\mathbf{X}^{(0)} = \bar{\mathbf{X}}^{(0)} = \mathbf{F}$ ,  $\mathbf{Y}^{(0)} = \bar{\mathbf{Y}}^{(0)} = \mathbf{0}$ ,  $\mathbf{P}^{(0)} = \mathbf{0}$ ,  $\mathbf{Q}^{(0)} = \mathbf{0}$ ,  $\sigma = \tau = \sqrt{\frac{2}{17 + \sqrt{33}}}$ 
2: for  $t = 1, \dots, n$  do
3:    $\mathbf{P}^{(t)} = \mathcal{P}_{\alpha_1}(\mathbf{P}^{(t-1)} + \sigma(\nabla \bar{\mathbf{X}}^{(t-1)} - \bar{\mathbf{Y}}^{(t-1)}))$ , where  $(\mathcal{P}_{\alpha_1}(\hat{\mathbf{P}}))_{i,j} = \hat{\mathbf{P}}_{i,j} / \max(1, \|\hat{\mathbf{P}}_{i,j}\|_2 / \alpha_1)$  ▷ Dual updates
4:    $\mathbf{Q}^{(t)} = \mathcal{P}_{\alpha_0}(\mathbf{Q}^{(t-1)} + \sigma \nabla \bar{\mathbf{Y}}^{(t-1)})$ , where  $(\mathcal{P}_{\alpha_0}(\hat{\mathbf{Q}}))_{i,j} = \hat{\mathbf{Q}}_{i,j} / \max(1, \|\hat{\mathbf{Q}}_{i,j}\|_2 / \alpha_0)$ 
5:    $\mathbf{X}^{(t)} = \frac{\theta \bar{\mathbf{X}}^{(t-1)} + \theta \tau \text{div}(\mathbf{P}^{(t)}) + \tau \mathbf{F}}{\theta + \tau}$  ▷ Primal updates
6:    $\mathbf{Y}^{(t)} = \mathbf{Y}^{(t-1)} + \tau(\mathbf{P}^{(t)} + \text{div}(\mathbf{Q}^{(t)}))$ 
7:    $\bar{\mathbf{X}}^{(t)} = 2\mathbf{X}^{(t)} - \bar{\mathbf{X}}^{(t-1)}$ 
8:    $\bar{\mathbf{Y}}^{(t)} = 2\mathbf{Y}^{(t)} - \bar{\mathbf{Y}}^{(t-1)}$ 
9: end for

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Figure 3: The primal-dual algorithm to solve $\text{argmin}_{\mathbf{X}} \frac{1}{2\theta} \|\mathbf{X} - \mathbf{F}\|_2^2 + \|\mathbf{X}\|_{\text{TGV}^2}$ in updating flow fields

In using TGV^2 as

$$\|\mathbf{X}\|_{\text{TGV}^2} = \min_{\mathbf{Y} \in \mathbb{R}^2} \left\{ \alpha_1 \int_{\Omega} |\nabla \mathbf{X} - \mathbf{Y}| d\mathbf{x} + \alpha_0 \int_{\Omega} |\nabla \mathbf{Y}| d\mathbf{x} \right\}, \quad (7)$$

we choose parameters α_1, α_0 to be $\alpha_1 = 1$ and $\alpha_0 = 5$ respectively as in [3].

References

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