Dense, Accurate Optical Flow Estimation with Piecewise Parametric Model (*Supplementary Material*)

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Abstract

In this supplementary material, we provide the details of the initialization algorithm, as well as more experimental results compared to state-of-the-art methods on the public benchmarks.

1. Initialization Algorithm

In this work, a simple strategy is used to generate candidate homography proposals and an initial labelling. We first compute an initial motion field via PatchMatch [1], then we use Direct Linear Transform (DLT) [3] to fit homographies for small local regions, and grow the regions to consistent neighbouring pixels for initial labelling. See Algorithm 3 for the details. In further we would like to test more initialization strategies.

2. More Results

2.1. Results on KITTI

Figure 1 shows the quantitative results on the the *test* set of the KITTI benchmark at the time of writing. Complete results can be found at the official webpage: http://www.cvlibs.net/datasets/kitti/eval_stereo_flow.php?benchmark=flow.

2.2. Results on Middlebury

Figure 2 shows the quantitative method evaluation results on the the *test* set of the Middlebury benchmark at the time of writing. Complete results can be found at the official webpage: http://vision.middlebury.edu/flow/eval/results/results-e1.php. Note that, all the methods have sub-pixel accuracy, and a very small difference in one sequence may lead to a large difference in ranking.

Algorithm 3: Homography proposal generation and initial labelling

Initialize a dense motion field by *e.g.* [1];
Initialize a label map with all pixels unlabelled;
$l \leftarrow 0$;
while *unlabelled pixels exist* **do**
    Pick out an unlabelled pixel $\mathbf{x}$;
    Fit a homography H_l with points in a small (*e.g.* 5×5) window $W_\mathbf{x}$ centered as $\mathbf{x}$;
    Label unlabelled pixels in $W_\mathbf{x}$ with l and push them into queue Q;
    while Q *is not empty* **do**
        Pop-out a pixel $\mathbf{p}$ from Q;
        foreach $\mathbf{q}$ *as* $\mathbf{p}$*'s unlabelled neighbour* **do**
            if $\mathbf{q}$*'s motion fits* H_l **then**
                Label $\mathbf{q}$ with l and push it into Q;
    $l \leftarrow l+1$;
if $l > L_{max}$ (e.g., *1000)* **then**
    Sort the labels according to their labelling areas;
    Set all pixels of the $l - L_{max}$ labels with smallest areas as unlabelled, then label each of them with its nearest label on the image.

Figure 3 compares the proposed method with method of [2] which uses translation and similarity models extracted from nearest neighbour fields. Visually inspected, our method yields smoother, and more accurate optical flow estimates.

We also show in Figure 4 the overlay of the reference frame and our optical flow estimation result on the on "Beanbags" and "DogDance" sequences.

2.3. Results on Sintel

Figure 5 shows the quantitative method evaluation results on the *test* set of the Sintel benchmark at the time of writing. Complete results can be found at the official webpage: `http://sintel.is.tue.mpg.de/results`.

References

[1] C. Barnes, E. Shechtman, A. Finkelstein, and D. Goldman. PatchMatch: A randomized correspondence algorithm for structural image editing. *ACM Transactions on Graphics (TOG)*, 28(3):24, 2009. 1

[2] Z. Chen, H. Jin, Z. Lin, S. Cohen, and Y. Wu. Large displacement optical flow from nearest neighbor fields. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 2443–2450, 2013. 1, 2

[3] R. Hartley and A. Zisserman. *Multiple view geometry in computer vision (2nd edition)*. Cambridge university press, 2005. 1

[4] J. Revaud, P. Weinzaepfel, Z. Harchaoui, and C. Schmid. EpicFlow: Edge-preserving interpolation of correspondences for optical flow. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2015. 5

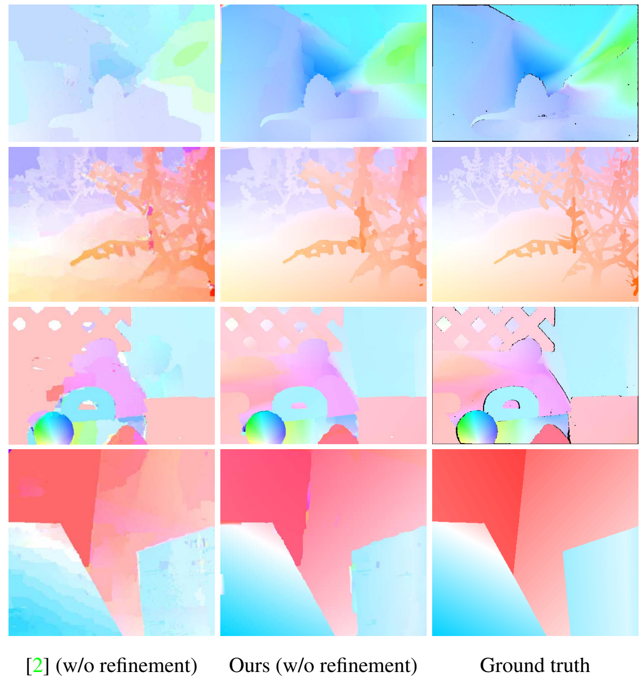

Figure 3: Qualitative comparison of [2] which uses global translation and similarity models (images reproduced from [2]), and our method. The flow fields shown here from both methods are without the refinement process.

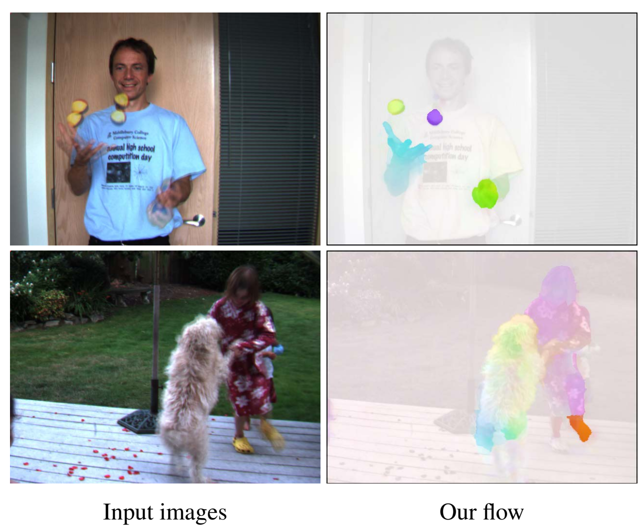

Figure 4: Results of our method on the “BeanBags” and “DogDance” sequences of Middlebury dataset.

Rank	Method	Setting	Code	Out-Noc	Out-All	Avg-Noc	Avg-All	Density	Runtime	Environment	Compare
1	VC-SF			**2.72 %**	**4.84 %**	0.8 px	1.3 px	100.00 %	300 s	1 core @ 2.5 Ghz (Matlab + C/C++)	
C. Vogel, S. Roth and K. Schindler: View-Consistent 3D Scene Flow Estimation over Multiple Frames. Proceedings of European Conference on Computer Vision. Lecture Notes in, Computer Science 2014.											
2	SPS-StFl			2.82 %	5.61 %	**0.8 px**	1.3 px	100.00 %	35 s	1 core @ 3.5 Ghz (C/C++)	
K. Yamaguchi, D. McAllester and R. Urtasun: Efficient Joint Segmentation, Occlusion Labeling, Stereo and Flow Estimation. ECCV 2014.											
3	SPS-Fl			3.38 %	10.06 %	0.9 px	2.9 px	100.00 %	11 s	1 core @ 3.5 Ghz (C/C++)	
K. Yamaguchi, D. McAllester and R. Urtasun: Efficient Joint Segmentation, Occlusion Labeling, Stereo and Flow Estimation. ECCV 2014.											
4	CVPR 1390			3.47 %	6.34 %	1.0 px	1.5 px	100.00 %	50 min	1 core @ 3.0 Ghz (Matlab + C/C++)	
Anonymous submission											
5	PR-Sf+E			3.57 %	7.07 %	0.9 px	1.6 px	100.00 %	200 s	4 cores @ 3.0 Ghz (Matlab + C/C++)	
C. Vogel, S. Roth and K. Schindler: Piecewise Rigid Scene Flow. International Conference on Computer Vision (ICCV) 2013.											
6	PCBP-Flow			3.64 %	8.28 %	0.9 px	2.2 px	100.00 %	3 min	4 cores @ 2.5 Ghz (Matlab + C/C++)	
K. Yamaguchi, D. McAllester and R. Urtasun: Robust Monocular Epipolar Flow Estimation. CVPR 2013.											
7	PR-Sceneflow			3.76 %	7.39 %	1.2 px	2.8 px	100.00 %	150 sec	4 core @ 3.0 Ghz (Matlab + C/C++)	
C. Vogel, S. Roth and K. Schindler: Piecewise Rigid Scene Flow. International Conference on Computer Vision (ICCV) 2013.											
8	MotionSLIC			3.91 %	10.56 %	0.9 px	2.7 px	100.00 %	11 s	1 core @ 3.0 Ghz (C/C++)	
K. Yamaguchi, D. McAllester and R. Urtasun: Robust Monocular Epipolar Flow Estimation. CVPR 2013.											
9	PPR-Flow			5.76 %	10.57 %	1.3 px	2.9 px	100.00 %	800 s	1 core @ 3.5 Ghz (Matlab + C/C++)	
Anonymous submission											
10	NLTGV-SC			5.93 %	11.96 %	1.6 px	3.8 px	100.00 %	16 s	GPU @ 2.5 Ghz (Matlab + C/C++)	
R. Ranftl, K. Bredies and T. Pock: Non-Local Total Generalized Variation for Optical Flow Estimation. Proceedings of the 13th European Conference on Computer Vision 2014.											
11	DDS-DF			6.03 %	13.08 %	1.6 px	4.2 px	100.00 %	1 min	1 core @ 2.5 Ghz (Matlab + C/C++)	
D. Wei, C. Liu and W. Freeman: A Data-driven Regularization Model for Stereo and Flow. 3DTV-Conference, 2014 International Conference on 2014.											
12	TGV2ADCSIFT			6.20 %	15.15 %	1.5 px	4.5 px	100.00 %	12s	GPU @ 2.4 Ghz (C/C++)	
J. Braux-Zin, R. Dupont and A. Bartoli: A General Dense Image Matching Framework Combining Direct and Feature-based Costs. International Conference on Computer Vision (ICCV) 2013.											
13	AnyFlow			6.37 %	15.80 %	1.5 px	4.3 px	100.00 %	15 s	GPU @ 2.5 Ghz (C/C++)	
Anonymous submission											
14	BTF-ILLUM			6.52 %	11.03 %	1.5 px	2.8 px	100.00 %	80 seconds	1 core @ 3.0 Ghz (C/C++)	
O. Demetz, M. Stoll, S. Volz, J. Weickert and A. Bruhn: Learning Brightness Transfer Functions for the Joint Recovery of Illumination Changes and Optical Flow. Computer Vision -- ECCV 2014 2014.											
15	CRT-TGV			6.71 %	12.09 %	2.0 px	3.9 px	100.00 %	10.5 min	1 core @ 3.0 Ghz (C/C++)	
Anonymous submission											
16	Data-Flow			7.11 %	14.57 %	1.9 px	5.5 px	100.00 %	3 min	2 cores @ 2.5 Ghz (Matlab + C/C++)	
C. Vogel, S. Roth and K. Schindler: An Evaluation of Data Costs for Optical Flow. German Conference on Pattern Recognition (GCPR) 2013.											
17	DeepFlow			7.22 %	17.79 %	1.5 px	5.8 px	100.00 %	17 s	1 core @ 3.6Ghz (Python + C/C++)	
P. Weinzaepfel, J. Revaud, Z. Harchaoui and C. Schmid: DeepFlow: Large displacement optical flow with deep matching. IEEE Intenational Conference on Computer Vision (ICCV) 2013.											
18	EpicFlow			7.88 %	17.08 %	1.5 px	3.8 px	100.00 %	15 s	1 core @ 3.6 Ghz (C/C++)	
Anonymous submission											
19	TVL1-HOG			7.91 %	18.90 %	2.0 px	6.1 px	100.00 %	180 s	2 cores @ 3.0 Ghz (Matlab)	
H. Rashwan, M. Mohamed, M. Garcia, B. Mertsching and D. Puig: Illumination Robust Optical Flow Model Based on Histogram of Oriented Gradients. German Conference on Pattern Recognition 2013 .											
20	MLDP-OF			8.67 %	18.78 %	2.4 px	6.7 px	100.00 %	160 s	2 cores @ 2.5 Ghz (Matlab)	
M. Mohamed, H. Rashwan, B. Mertsching, M. Garcia and D. Puig: Illumination-Robust Optical Flow Using Local Directional Pattern . IEEE Transactions on Circuits and Systems for Video Technology 2014 .											
21	DescFlow			8.76 %	19.45 %	2.1 px	5.7 px	100.00 %	9.0 s	GPU @ 2.5 Ghz (C/C++)	
Anonymous submission											
22	SparseFlow		code	9.09 %	19.32 %	2.6 px	7.6 px	100.00 %	10 s	1 core @ 3.5 Ghz (Matlab + C/C++)	
R. Timofte and L. Gool: SparseFlow: Sparse Matching for Small to Large Displacement Optical Flow . WACV 2015 .											
23	CRTflow			9.43 %	18.72 %	2.7 px	6.5 px	100.00 %	18 s	GPU @ 1.0 Ghz (C/C++)	
O. Demetz, D. Hafner and J. Weickert: The Complete Rank Transform: A Tool for Accurate and Morphologically Invariant Matching of Structure. Proc.~British Machine Vision Conference 2013 (BMVC) 2013											
24	C++		code	10.04 %	20.26 %	2.6 px	7.1 px	100.00 %	8.5 min	1 core @ 3.0 Ghz (Matlab)	
D. Sun, S. Roth and M. Black: A Quantitative Analysis of Current Practices in Optical Flow Estimation and The Principles Behind Them. 2014.											
25	TF+OFM		code	10.22 %	18.46 %	2.0 px	5.0 px	100.00 %	350 s	1 cores @ 2.5 Ghz (Matlab + C/C++)	
R. Kennedy and C. Taylor: Optical Flow with Geometric Occlusion Estimation and Fusion of Multiple Frames. EMMCVPR 2015.											
26	C+NL		code	10.49 %	20.64 %	2.8 px	7.2 px	100.00 %	14.8 min	1 core @ 3.0 Ghz (Matlab)	
D. Sun, S. Roth and M. Black: A Quantitative Analysis of Current Practices in Optical Flow Estimation and The Principles Behind Them. 2014.											
27	NNF-Local			10.68 %	21.09 %	2.7 px	7.4 px	100.00 %	1073 s	1 core @ 2.5 Ghz (Matlab)	
28	fSGM			10.74 %	22.66 %	3.2 px	12.2 px	100.00 %	60 s	1 core @ 2.4 Ghz (C/C++)	
S. Hermann and R. Klette: Hierarchical Scan Line Dynamic Programming for Optical Flow using Semi-Global Matching. ACCV Workshops 2012.											
29	TGV2CENSUS		code	11.03 %	18.37 %	2.9 px	6.6 px	100.00 %	4 s	GPU+CPU @ 3.0 Ghz (Matlab + C/C++)	
M. Werlberger: Convex Approaches for High Performance Video Processing. 2012. R. Ranftl, S. Gehrig, T. Pock and H. Bischof: Pushing the Limits of Stereo Using Variational Stereo Estimation. IV 2012.											
30	C+NL-fast		code	12.36 %	22.28 %	3.2 px	7.9 px	100.00 %	2.9 min	1 core @ 3.0 Ghz (Matlab)	
D. Sun, S. Roth and M. Black: A Quantitative Analysis of Current Practices in Optical Flow Estimation and The Principles Behind Them. 2014.											

Figure 1: Method evaluation on KITTI benchmark with the default 3-pixel error threshold (captured on 21-Nov-2014). Our method "PPR-Flow" (new name "PH-Flow" now) ranks 1st among all pure optical flow methods without stereo information or epipolar constraint.

Average endpoint error	avg.	Army (Hidden texture) GT im0 im1			Mequon (Hidden texture) GT im0 im1			Schefflera (Hidden texture) GT im0 im1			Wooden (Hidden texture) GT im0 im1			Grove (Synthetic) GT im0 im1			Urban (Synthetic) GT im0 im1			Yosemite (Synthetic) GT im0 im1			Teddy (Stereo) GT im0 im1		
	rank	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext
NNF-Local [87]	2.7	0.07 1	0.20 2	0.05 1	0.15 1	0.51 3	0.12 5	0.18 1	0.37 1	0.14 1	0.10 2	0.49 3	0.06 2	0.41 1	0.61 1	0.21 2	0.23 2	0.66 2	0.19 1	0.10 4	0.12 9	0.17 12	0.34 1	0.80 4	0.23 2
OFLAF [77]	7.0	0.08 7	0.21 3	0.06 5	0.16 5	0.53 4	0.12 5	0.19 2	0.37 1	0.14 1	0.14 7	0.77 23	0.07 4	0.51 4	0.78 5	0.25 3	0.31 6	0.76 3	0.25 8	0.11 11	0.12 9	0.21 31	0.42 7	0.78 2	0.63 12
MDP-Flow2 [68]	8.2	0.08 7	0.21 3	0.07 14	0.15 1	0.48 1	0.11 1	0.20 4	0.40 4	0.14 1	0.15 18	0.80 29	0.08 10	0.63 16	0.93 16	0.43 17	0.26 3	0.76 3	0.23 6	0.11 11	0.12 9	0.17 12	0.38 3	0.79 3	0.44 4
NN-field [71]	9.0	0.08 7	0.22 14	0.05 1	0.17 7	0.55 7	0.13 10	0.19 2	0.39 3	0.15 6	0.09 1	0.48 2	0.05 1	0.41 1	0.61 1	0.20 1	0.52 48	0.64 1	0.26 11	0.13 32	0.13 27	0.20 25	0.35 2	0.83 5	0.21 1
ComponentFusion [96]	10.6	0.07 1	0.21 3	0.05 1	0.16 5	0.55 7	0.12 5	0.20 4	0.44 7	0.15 6	0.11 3	0.65 6	0.06 2	0.71 29	1.07 34	0.53 31	0.32 8	1.06 22	0.28 14	0.11 11	0.13 27	0.15 7	0.41 6	0.88 10	0.54 5
TC/T-Flow [76]	15.8	0.07 1	0.21 3	0.05 1	0.19 15	0.68 28	0.12 5	0.28 20	0.66 25	0.14 1	0.14 7	0.86 38	0.07 4	0.67 26	0.98 25	0.49 26	0.22 1	0.82 7	0.19 1	0.11 11	0.11 1	0.30 70	0.50 23	1.02 26	0.64 14
WLIF-Flow [93]	15.8	0.08 7	0.21 3	0.06 5	0.18 10	0.55 7	0.15 20	0.25 15	0.56 16	0.17 13	0.14 7	0.68 7	0.08 10	0.61 14	0.91 15	0.41 15	0.43 25	0.96 13	0.29 20	0.13 32	0.12 9	0.21 31	0.51 28	1.03 29	0.72 28
NNF-EAC [104]	16.8	0.09 29	0.22 14	0.07 14	0.17 7	0.53 4	0.13 10	0.23 9	0.49 10	0.15 6	0.16 30	0.80 29	0.09 23	0.60 11	0.89 11	0.40 13	0.38 16	0.78 5	0.28 14	0.12 22	0.12 9	0.18 17	0.57 37	1.24 41	0.69 23
Layers++ [37]	17.3	0.08 7	0.21 3	0.07 14	0.19 15	0.56 10	0.17 27	0.20 4	0.40 4	0.18 19	0.13 6	0.58 4	0.07 4	0.48 3	0.70 3	0.33 6	0.47 36	1.01 16	0.33 37	0.15 54	0.14 49	0.24 43	0.46 13	0.88 10	0.72 28
LME [70]	17.8	0.08 7	0.22 14	0.06 5	0.15 1	0.49 2	0.11 1	0.30 28	0.64 20	0.31 71	0.15 18	0.78 26	0.09 23	0.66 22	0.96 21	0.53 31	0.33 9	1.18 34	0.28 14	0.12 22	0.12 9	0.18 17	0.44 9	0.91 12	0.61 10
IROF++ [58]	18.4	0.08 7	0.23 20	0.07 14	0.21 27	0.68 28	0.17 27	0.28 20	0.63 19	0.19 31	0.15 18	0.73 19	0.09 23	0.60 11	0.89 11	0.42 16	0.43 25	1.08 25	0.31 27	0.10 4	0.12 9	0.12 4	0.47 15	0.98 19	0.68 22
nLayers [57]	18.6	0.07 1	0.19 1	0.06 5	0.22 33	0.59 13	0.19 46	0.25 15	0.54 13	0.20 40	0.15 18	0.84 35	0.08 10	0.53 5	0.78 5	0.34 8	0.44 29	0.84 8	0.30 24	0.13 32	0.13 27	0.20 25	0.47 15	0.97 18	0.67 20
FC-2Layers-FF [74]	20.5	0.08 7	0.21 3	0.07 14	0.21 27	0.70 33	0.17 27	0.20 4	0.40 4	0.18 19	0.15 18	0.76 22	0.08 10	0.53 5	0.77 4	0.37 9	0.49 42	1.02 17	0.33 37	0.16 65	0.13 27	0.29 65	0.44 9	0.87 9	0.64 14
PPR-Flow [102]	20.5	0.08 7	0.24 27	0.07 14	0.21 27	0.68 28	0.17 27	0.23 9	0.49 10	0.19 31	0.16 30	0.83 33	0.09 23	0.56 7	0.83 7	0.38 10	0.30 5	0.81 6	0.24 7	0.15 54	0.13 27	0.30 70	0.43 8	0.85 6	0.66 18
Correlation Flow [75]	21.0	0.09 29	0.23 20	0.07 14	0.17 7	0.58 12	0.11 1	0.43 49	0.99 51	0.15 6	0.11 3	0.47 1	0.08 10	0.75 35	1.08 35	0.56 36	0.41 21	0.92 11	0.30 24	0.14 42	0.13 27	0.27 56	0.40 5	0.85 6	0.42 3
AGIF+OF [85]	22.1	0.08 7	0.22 14	0.07 14	0.23 45	0.73 37	0.18 37	0.28 20	0.66 25	0.18 19	0.14 7	0.70 10	0.08 10	0.57 8	0.85 8	0.38 10	0.47 36	0.97 14	0.31 27	0.13 32	0.13 27	0.22 36	0.51 28	0.99 22	0.74 37
FESL [72]	24.0	0.08 7	0.21 3	0.07 14	0.25 55	0.75 43	0.19 46	0.27 17	0.61 17	0.18 19	0.14 7	0.68 7	0.08 10	0.61 14	0.89 11	0.44 18	0.47 36	1.03 20	0.32 32	0.14 42	0.15 59	0.25 48	0.50 23	0.96 16	0.63 12
Classic+CPF [83]	24.0	0.08 7	0.23 20	0.07 14	0.22 33	0.73 37	0.17 27	0.30 28	0.70 28	0.18 19	0.14 7	0.72 18	0.08 10	0.63 16	0.93 16	0.45 20	0.51 46	1.03 20	0.32 32	0.14 42	0.12 9	0.30 70	0.48 17	0.93 13	0.72 28
ALD-Flow [66]	24.1	0.07 1	0.21 3	0.06 5	0.19 15	0.64 22	0.13 10	0.30 28	0.73 31	0.15 6	0.17 37	0.92 50	0.07 4	0.78 38	1.14 39	0.59 39	0.33 9	1.30 42	0.21 4	0.12 22	0.12 9	0.28 60	0.54 33	1.19 39	0.73 33
TC-Flow [46]	24.4	0.07 1	0.21 3	0.06 5	0.15 1	0.59 13	0.11 1	0.31 33	0.78 36	0.14 1	0.16 30	0.86 38	0.08 10	0.75 35	1.11 37	0.54 33	0.42 23	1.40 51	0.25 8	0.11 11	0.12 9	0.29 65	0.62 42	1.35 43	0.93 57
COFM [59]	24.5	0.08 7	0.26 38	0.06 5	0.18 10	0.62 18	0.14 16	0.30 28	0.74 33	0.19 31	0.15 18	0.86 38	0.07 4	0.79 39	1.14 39	0.74 56	0.35 14	0.87 10	0.28 14	0.14 42	0.12 9	0.28 60	0.49 19	0.94 14	0.71 27
Efficient-NL [60]	24.8	0.08 7	0.22 14	0.06 5	0.21 27	0.67 26	0.17 27	0.31 33	0.73 31	0.18 19	0.14 7	0.71 15	0.08 10	0.59 10	0.88 10	0.39 12	1.30 79	1.35 46	0.67 74	0.14 42	0.13 27	0.26 50	0.45 11	0.85 6	0.55 7
Sparse-NonSparse [56]	24.9	0.08 7	0.23 20	0.07 14	0.22 33	0.73 37	0.18 37	0.28 20	0.64 20	0.19 31	0.14 7	0.71 15	0.08 10	0.67 26	0.99 27	0.48 23	0.49 42	1.06 22	0.32 32	0.14 42	0.11 1	0.28 60	0.49 19	0.98 19	0.73 33
LSM [39]	26.5	0.08 7	0.23 20	0.07 14	0.22 33	0.73 37	0.18 37	0.28 20	0.64 20	0.19 31	0.14 7	0.70 10	0.09 23	0.66 22	0.97 23	0.48 23	0.50 44	1.06 22	0.33 37	0.15 54	0.12 9	0.29 65	0.50 23	0.99 22	0.73 33
Ramp [62]	27.0	0.08 7	0.24 27	0.07 14	0.21 27	0.72 35	0.18 37	0.27 17	0.62 18	0.19 31	0.15 18	0.71 15	0.09 23	0.66 22	0.97 23	0.49 26	0.51 46	1.09 26	0.34 43	0.15 54	0.12 9	0.30 70	0.48 17	0.96 16	0.72 28
Classic+NL [31]	28.9	0.08 7	0.23 20	0.07 14	0.22 33	0.74 41	0.18 37	0.29 25	0.65 24	0.19 31	0.15 18	0.73 19	0.09 23	0.64 19	0.93 16	0.47 21	0.52 48	1.12 29	0.33 37	0.16 65	0.13 27	0.29 65	0.49 19	0.98 19	0.74 37
TV-L1-MCT [64]	29.3	0.08 7	0.23 20	0.07 14	0.24 50	0.77 46	0.19 46	0.32 36	0.76 35	0.19 31	0.14 7	0.69 9	0.09 23	0.72 31	1.03 28	0.60 40	0.54 51	1.10 27	0.35 46	0.11 11	0.12 9	0.20 25	0.54 33	1.04 31	0.84 48
PMF [73]	30.0	0.09 29	0.25 32	0.07 14	0.19 15	0.60 16	0.14 16	0.23 9	0.46 9	0.17 13	0.17 37	0.87 42	0.09 23	0.58 9	0.86 9	0.26 4	0.82 66	1.17 32	0.54 64	0.21 89	0.22 94	0.36 85	0.39 4	0.75 1	0.59 9
FMOF [94]	31.0	0.08 7	0.22 14	0.07 14	0.24 50	0.76 44	0.19 46	0.24 12	0.54 13	0.18 19	0.14 7	0.70 10	0.08 10	0.64 19	0.94 20	0.44 18	1.19 75	1.12 29	0.65 73	0.15 54	0.13 27	0.32 80	0.58 39	1.16 37	0.70 26
IROF-TV [53]	32.3	0.09 29	0.25 32	0.08 38	0.22 33	0.77 46	0.19 46	0.30 28	0.70 28	0.19 31	0.18 44	0.93 53	0.11 46	0.73 33	1.04 30	0.56 36	0.44 29	1.69 70	0.31 27	0.09 3	0.11 1	0.12 4	0.50 23	1.08 33	0.73 33
MDP-Flow [26]	33.5	0.09 29	0.25 32	0.08 38	0.19 15	0.54 6	0.18 37	0.24 12	0.55 15	0.20 40	0.16 30	0.91 47	0.09 23	0.74 34	1.06 33	0.61 42	0.46 33	1.02 17	0.35 46	0.12 22	0.14 49	0.17 12	0.78 64	1.68 67	0.97 62
2DHMM-SAS [92]	34.5	0.08 7	0.24 27	0.07 14	0.23 45	0.78 49	0.17 27	0.42 48	0.90 43	0.22 50	0.15 18	0.75 21	0.09 23	0.65 21	0.96 21	0.48 23	0.56 54	1.13 31	0.34 43	0.15 54	0.13 27	0.30 70	0.56 36	1.13 35	0.79 41
MLDP_OF [89]	34.8	0.11 48	0.28 47	0.09 52	0.18 10	0.56 10	0.13 10	0.34 38	0.79 38	0.17 13	0.16 30	0.82 32	0.09 23	0.72 31	1.05 32	0.50 28	0.34 12	1.10 27	0.27 13	0.18 77	0.15 59	0.44 92	0.76 57	1.09 34	0.69 23
EPPM w/o HM [88]	35.0	0.11 48	0.30 58	0.08 38	0.19 15	0.67 26	0.13 10	0.29 25	0.71 30	0.17 13	0.17 37	0.78 26	0.11 46	0.63 16	0.93 16	0.33 6	0.60 57	1.35 46	0.40 56	0.19 80	0.15 59	0.45 93	0.45 11	0.94 14	0.64 14
Sparse Occlusion [54]	35.7	0.09 29	0.24 27	0.08 38	0.22 33	0.63 19	0.19 46	0.38 43	0.91 44	0.18 19	0.17 37	0.85 37	0.09 23	0.75 35	1.09 36	0.47 21	0.34 12	1.00 15	0.26 11	0.22 91	0.22 94	0.28 60	0.53 32	1.13 35	0.67 20
OFH [38]	35.8	0.10 41	0.25 32	0.09 52	0.19 15	0.69 31	0.14 16	0.43 49	1.02 55	0.17 13	0.17 37	1.08 60	0.08 10	0.87 49	1.25 47	0.73 53	0.43 25	1.69 70	0.32 32	0.10 4	0.13 27	0.18 17	0.59 40	1.40 47	0.74 37
NL-TV-NCC [25]	36.8	0.10 41	0.26 38	0.08 38	0.22 33	0.72 35	0.15 20	0.35 39	0.85 40	0.16 11	0.15 18	0.70 10	0.09 23	0.79 39	1.16 42	0.51 29	0.78 64	1.38 48	0.48 61	0.16 65	0.15 59	0.26 50	0.55 35	1.16 37	0.55 7
CostFilter [40]	36.9	0.10 41	0.27 45	0.08 38	0.20 25	0.63 19	0.15 20	0.22 8	0.45 8	0.18 19	0.19 48	0.88 44	0.12 52	0.60 11	0.90 14	0.28 5	0.75 63	1.19 35	0.50 62	0.21 89	0.24 100	0.40 89	0.46 13	1.02 26	0.62 11
S2D-Matching [84]	37.0	0.09 29	0.26 38	0.07 14	0.23 45	0.80 54	0.18 37	0.38 43	0.93 46	0.20 40	0.15 18	0.70 10	0.09 23	0.70 28	1.03 28	0.51 29	0.55 53	1.17 32	0.35 46	0.17 72	0.13 27	0.32 80	0.51 28	1.01 24	0.81 44
AggregFlow [97]	38.0	0.11 48	0.32 64	0.08 38	0.31 69	0.96 71	0.23 67	0.36 41	0.85 40	0.27 63	0.17 37	0.84 35	0.10 42	0.79 39	1.17 43	0.54 33	0.27 4	0.85 9	0.19 1	0.11 11	0.13 27	0.15 7	0.59 40	1.19 39	0.83 45
SimpleFlow [49]	39.5	0.09 29	0.24 27	0.08 38	0.24 50	0.78 49	0.20 57	0.43 49	0.96 49	0.21 45	0.16 30	0.77 23	0.09 23	0.71 29	1.04 30	0.55 35	1.47 86	1.59 64	0.76 77	0.13 32	0.12 9	0.22 36	0.50 23	1.04 31	0.72 28
Aniso-Texture [82]	40.0	0.08 7	0.21 3	0.07 14	0.19 15	0.60 16	0.17 27	0.50 59	1.11 60	0.21 45	0.12 5	0.58 4	0.07 4	0.93 62	1.28 55	0.92 67	0.46 33	1.27 39	0.38 55	0.20 82	0.20 91	0.30 70	0.68 48	1.37 45	0.88 53
Occlusion-TV-L1 [63]	41.2	0.09 29	0.26 38	0.07 14	0.22 33	0.74 41	0.18 37	0.51 61	1.15 64	0.21 45	0.18 44	0.91 47	0.10 42	0.87 49	1.25 47	0.72 50	0.47 36	1.38 48	0.36 50	0.10 4	0.12 9	0.11 2	0.83 67	1.78 70	0.96 61
Adaptive [20]	44.1	0.09 29	0.26 38	0.06 5	0.23 45	0.78 49	0.18 37	0.54 65	1.19 70	0.21 45	0.18 44	0.91 47	0.10 42	0.88 52	1.25 47	0.73 53	0.50 44	1.28 40	0.31 27	0.14 42	0.16 69	0.22 36	0.65 46	1.37 45	0.79 41
SRR-TVOF-NL [91]	44.7	0.11 48	0.29 53	0.08 38	0.28 64	0.91 66	0.20 57	0.39 45	0.92 45	0.24 55	0.17 37	0.77 23	0.09 23	0.81 42	1.11 37	0.79 58	0.33 9	1.02 17	0.28 14	0.19 80	0.18 83	0.31 77	0.57 37	1.01 24	0.77 40
RFlow [90]	45.4	0.10 41	0.27 45	0.09 52	0.19 15	0.64 22	0.15 20	0.46 57	1.07 56	0.18 19	0.22 61	1.31 73	0.11 46	0.92 60	1.30 58	0.91 66	0.42 23	1.42 54	0.31 27	0.14 42	0.13 27	0.24 43	0.77 61	1.66 64	0.94 58
TCOF [69]	45.5	0.11 48	0.28 47	0.09 52	0.24 50	0.76 44	0.19 46	0.53 62	1.15 64	0.29 67	0.24 63	0.88 44	0.20 74	0.88 52	1.26 51	0.69 46	0.38 16	0.93 12	0.29 20	0.16 65	0.16 69	0.22 36	0.49 19	1.03 29	0.65 17
DPOF [18]	45.6	0.12 66	0.33 67	0.08 38	0.26 59	0.80 54	0.20 57	0.24 12	0.49 10	0.20 40	0.19 48	0.83 33	0.13 56	0.66 22	0.98 25	0.40 13	1.11 74	1.41 53	0.57 68	0.25 96	0.14 49	0.55 96	0.51 28	1.02 26	0.54 5
Complementary OF [21]	46.0	0.11 48	0.28 47	0.10 66	0.18 10	0.63 19	0.12 5	0.31 33	0.75 34	0.18 19	0.19 48	0.97 54	0.12 52	0.97 67	1.31 62	1.00 73	1.78 95	1.73 73	0.87 85	0.11 11	0.12 9	0.22 36	0.68 48	1.48 51	0.95 59
ACK-Prior [27]	46.2	0.11 48	0.25 32	0.09 52	0.18 10	0.59 13	0.13 10	0.27 17	0.64 20	0.16 11	0.15 18	0.78 26	0.09 23	0.82 44	1.14 39	0.71 49	1.90 96	1.90 79	0.99 92	0.23 94	0.17 76	0.49 95	0.77 61	1.44 50	0.91 55
EpicFlow [103]	47.4	0.12 66	0.36 77	0.09 52	0.25 55	0.85 60	0.21 62	0.39 45	1.00 52	0.25 57	0.19 48	1.01 56	0.11 46	0.89 54	1.31 62	0.69 46	0.53 50	1.31 43	0.34 43	0.10 4	0.11 1	0.17 12	0.67 47	1.43 49	0.87 51
ComplOF-FED-GPU [35]	48.4	0.11 48	0.29 53	0.10 66	0.21 27	0.78 49	0.14 16	0.32 36	0.79 38	0.17 13	0.19 48	0.99 55	0.11 46	0.89 54	1.29 56	0.73 53	1.25 77	1.74 74	0.64 72	0.14 42	0.13 27	0.30 70	0.64 44	1.50 53	0.83 45
Classic++ [32]	49.8	0.09 29	0.25 32	0.07 14	0.23 45	0.78 49	0.19 46	0.43 49	1.00 52	0.22 50	0.20 53	1.11 61	0.10 42	0.87 49	1.30 58	0.66 45	0.47 36	1.62 65	0.33 37	0.17 72	0.14 49	0.32 80	0.79 65	1.64 62	0.92 56
Aniso. Huber-L1 [22]	50.5	0.10 41	0.28 47	0.08 38	0.31 69	0.88 63	0.28 74	0.56 68	1.13 61	0.29 67	0.20 53	0.92 50	0.13 56	0.84 46	1.20 44	0.70 48	0.39 19	1.23 37	0.28 14	0.17 72	0.15 59	0.27 56	0.64 44	1.36 44	0.79 41
TF+OM [100]	51.7	0.10 41	0.26 38	0.07 14	0.22 33	0.66 25	0.19 46	0.36 41	0.78 36	0.39 75	0.20 53	0.89 46	0.13 56	0.98 70	1.31 62	1.03 74	0.56 54	1.55 62	0.33 37	0.16 65	0.17 76	0.27 56	0.76 57	1.59 60	0.98 63
DeepFlow [86]	52.5	0.12 66	0.31 61	0.11 73	0.28 64	0.82 57	0.22 65	0.44 55	1.00 52	0.33 72	0.26 69	1.34 76	0.15 65	0.81 42	1.21 45	0.58 38	0.38 16	1.55 62	0.25 8	0.11 11	0.11 1	0.24 43	0.93 73	1.82 74	1.12 71
CRTflow [80]	52.5	0.11 48	0.30 58	0.08 38	0.24 50	0.77 46	0.17 27	0.50 59	1.13 61	0.21 45	0.23 62	1.24 68	0.12 52	0.86 48	1.27 53	0.65 44	0.60 57	1.95 83	0.50 62	0.12 22	0.14 49	0.21 31	0.79 65	1.77 69	0.98 63
TriangleFlow [30]	53.2	0.11 48	0.29 53	0.09 52	0.26 59	0.95 69	0.17 27	0.47 58	1.07 56	0.18 19	0.16 30	0.87 42	0.09 23	1.07 77	1.47 83	1.10 79	0.87 67	1.39 50	0.57 68	0.15 54	0.19 89	0.23 42	0.63 43	1.33 42	0.84 48
TV-L1-improved [17]	55.4	0.09 29	0.26 38	0.07 14	0.20 25	0.71 34	0.16 24	0.53 62	1.18 69	0.22 50	0.21 58	1.24 68	0.11 46	0.90 56	1.31 62	0.72 50	1.51 88	1.93 81	0.84 81	0.18 77	0.17 76	0.31 77	0.73 53	1.62 61	0.87 51
SIOF [67]	55.7	0.11 48	0.28 47	0.09 52	0.27 62	0.95 69	0.20 57	0.60 75	1.17 66	0.48 76	0.25 67	1.13 62	0.16 66	0.97 67	1.33 67	1.03 74	0.43 25	1.32 44	0.36 50	0.13 32	0.13 27	0.18 17	0.76 57	1.52 55	1.14 75
LocallyOriented [52]	57.4	0.12 66	0.35 72	0.08 38	0.33 73	1.01 74	0.25 70	0.61 77	1.30 78	0.28 64	0.18 44	0.80 29	0.13 56	0.93 62	1.29 56	0.79 58	0.98 70	1.48 58	0.56 67	0.12 22	0.14 49	0.21 31	0.73 53	1.48 51	0.95 59
CBF [12]	57.5	0.10 41	0.28 47	0.09 52	0.34 74	0.80 54	0.37 78	0.43 49	0.95 48	0.26 59	0.21 58	1.14 63	0.13 56	0.90 56	1.27 53	0.82 61	0.41 21	1.23 37	0.30 24	0.23 94	0.19 89	0.39 88	0.76 57	1.56 56	1.02 65
Brox et al. [5]	59.6	0.11 48	0.32 64	0.11 73	0.27 62	0.93 67	0.22 65	0.39 45	0.94 47	0.24 55	0.24 63	1.25 70	0.13 56	1.10 83	1.39 77	1.43 92	0.89 69	1.77 76	0.55 66	0.10 4	0.13 27	0.11 2	0.91 70	1.83 76	1.13 73
Local-TV-L1 [65]	59.8	0.14 76	0.34 69	0.14 81	0.47 81	1.05 77	0.43 81	0.72 82	1.25 75	0.52 77	0.31 77	1.39 79	0.22 76	0.83 45	1.21 45	0.63 43	0.39 19	1.29 41	0.29 20	0.11 11	0.11 1	0.22 36	1.06 78	1.87 77	1.67 87
F-TV-L1 [15]	60.0	0.14 76	0.35 72	0.14 81	0.34 74	0.98 72	0.26 71	0.59 74	1.19 70	0.26 59	0.27 72	1.36 78	0.16 66	0.90 56	1.30 58	0.76 57	0.54 51	1.62 65	0.36 50	0.13 32	0.15 59	0.20 25	0.68 48	1.56 56	0.66 18
CLG-TV [48]	60.2	0.11 48	0.29 53	0.09 52	0.32 71	0.86 62	0.30 75	0.55 66	1.17 66	0.28 64	0.25 67	1.05 58	0.17 69	0.92 60	1.30 58	0.79 58	0.47 36	1.72 72	0.35 46	0.17 72	0.17 76	0.25 48	0.74 56	1.57 58	0.88 53
Fusion [6]	61.3	0.11 48	0.34 69	0.10 66	0.19 15	0.69 31	0.16 24	0.29 25	0.66 25	0.23 53	0.20 53	1.19 65	0.14 63	1.07 77	1.42 80	1.22 85	1.35 80	1.49 59	0.86 83	0.20 82	0.20 91	0.26 50	1.07 80	2.07 86	1.39 82
Rannacher [23]	61.4	0.11 48	0.31 61	0.09 52	0.25 55	0.84 59	0.21 62	0.57 71	1.27 77	0.26 59	0.24 63	1.32 74	0.13 56	0.91 59	1.33 67	0.72 50	1.49 87	1.95 83	0.78 78	0.15 54	0.14 49	0.26 50	0.69 51	1.58 59	0.86 50
SuperFlow [81]	61.7	0.11 48	0.29 53	0.08 38	0.34 74	0.85 60	0.33 76	0.53 62	1.08 59	0.59 80	0.28 74	1.23 67	0.21 75	0.99 71	1.32 66	1.21 84	0.46 33	1.49 59	0.36 50	0.15 54	0.16 69	0.19 21	0.90 69	1.81 72	1.07 67
TriFlow [95]	63.4	0.12 66	0.33 67	0.09 52	0.30 68	0.89 64	0.27 72	0.56 68	1.17 66	0.57 79	0.21 58	0.92 50	0.16 66	1.07 77	1.38 73	1.19 83	0.35 14	1.19 35	0.29 20	0.52 102	0.22 94	1.30 102	0.73 53	1.42 48	0.83 45

Figure 2: Method evaluation on Middlebury benchmark with average end-point error (captured on 21-Nov-2014). Our method “PPR-Flow” (new name “PH-Flow” now) ranks 13rd among all the methods.

	EPE all	EPE matched	EPE unmatched	d0-10	d10-60	d60-140	s0-10	s10-40	s40+
EpicFlow [2]	4.115	1.360	26.595	3.660	1.079	0.599	0.712	2.117	25.859
PPR-Flow [3]	4.388	1.714	26.202	3.612	1.713	0.834	0.590	2.430	27.997
AggregFlow [4]	4.754	1.694	29.685	3.705	1.603	0.981	0.650	2.251	31.184
TF+OFM [5]	4.917	1.874	29.735	3.676	1.689	1.309	0.839	2.349	31.391
SparseFlowFused [6]	5.257	1.627	34.834	4.211	1.397	0.729	0.880	2.567	33.489
DeepFlow [7]	5.377	1.771	34.751	4.519	1.534	0.837	0.960	2.730	33.701
NNF-Local [8]	5.386	1.397	37.896	2.722	1.341	1.004	0.683	2.245	36.342
PatchWMF-OF [9]	5.550	1.781	36.257	3.339	1.843	1.277	0.581	2.612	37.319
WLIF-Flow [10]	5.734	1.759	38.125	3.242	1.818	1.296	0.597	2.512	39.036
AGIF+OF [11]	5.766	1.695	38.936	3.034	1.709	1.329	0.613	2.554	39.121
CVPR-738-Multi [12]	5.800	2.559	32.263	5.651	2.489	1.428	1.202	3.200	34.939
LocalLayering [13]	5.820	2.143	35.784	3.817	2.342	1.399	0.580	2.461	39.976
MDP-Flow2 [14]	5.837	1.869	38.158	3.210	1.913	1.441	0.640	2.603	39.459
ComponentFusion [15]	6.065	2.033	38.912	4.114	2.063	1.213	0.910	2.996	39.074
AnyFlow [16]	6.066	2.412	35.852	5.211	2.432	1.215	1.429	3.665	34.900
SparseFlow [17]	6.197	2.357	37.460	4.642	2.273	1.392	0.681	2.533	42.422
EPPM [18]	6.494	2.675	37.632	4.997	2.422	1.948	1.402	3.446	39.152
S2D-Matching [19]	6.510	2.792	36.785	5.523	3.018	1.546	0.622	3.012	44.187
Classic+NLP [20]	6.731	2.949	37.545	5.573	3.291	1.648	0.638	3.296	45.290
FC-2Layers-FF [21]	6.781	3.053	37.144	5.841	3.390	1.688	0.580	3.308	45.962

(a) Results on the “Clean” sequences. Our method “PPR-Flow” (new name “PH-Flow” now) ranks 2nd among all evaluated methods (method “EpicFlow” [4] was unpublished at the time of writing).

	EPE all	EPE matched	EPE unmatched	d0-10	d10-60	d60-140	s0-10	s10-40	s40+
EpicFlow [2]	6.285	3.060	32.564	5.205	2.611	2.216	1.135	3.727	38.021
TF+OFM [3]	6.727	3.388	33.929	5.544	3.238	2.551	1.512	3.765	39.761
SparseFlowFused [4]	7.189	3.286	38.977	5.567	3.098	2.159	1.275	3.963	44.319
DeepFlow [5]	7.212	3.336	38.781	5.650	3.144	2.208	1.284	4.107	44.118
NNF-Local [6]	7.249	2.973	42.088	4.896	2.817	2.218	1.159	4.183	44.866
AggregFlow [7]	7.329	3.696	36.929	5.538	3.435	2.918	1.241	4.296	44.858
PPR-Flow [8]	7.423	3.795	36.960	5.550	3.675	2.716	1.119	4.827	44.926
SparseFlow [9]	7.851	3.855	40.401	6.117	3.838	2.557	1.071	3.771	51.353
S2D-Matching [10]	7.872	3.918	40.093	5.975	3.815	2.851	1.172	4.695	48.782
AnyFlow [11]	7.933	3.994	40.027	6.284	3.997	2.756	2.279	5.391	42.122
PatchWMF-OF [12]	7.971	3.766	42.218	5.712	3.568	2.797	1.279	4.970	48.396
LocalLayering [13]	8.043	4.014	40.879	5.680	3.841	3.122	1.186	4.990	49.426
WLIF-Flow [14]	8.049	3.837	42.348	5.851	3.657	2.811	1.290	5.033	48.843
FC-2Layers-FF [15]	8.137	4.261	39.723	6.537	4.257	2.946	1.034	4.835	51.349
ComponentFusion [16]	8.231	4.274	40.460	6.221	4.252	3.193	1.702	5.701	46.696
CVPR-738-Multi [17]	8.235	4.660	37.397	7.596	4.642	3.133	1.976	4.630	48.019
MLDP-OF [18]	8.287	4.165	41.905	6.345	4.127	2.996	1.312	5.122	50.540
Classic+NLP [19]	8.291	4.287	40.925	6.520	4.265	2.984	1.208	5.090	51.162
EPPM [20]	8.377	4.286	41.695	6.556	4.024	3.323	1.834	4.955	49.083
MDP-Flow2 [21]	8.445	4.150	43.430	5.703	3.925	3.406	1.420	5.449	50.507

(b) Results on the “Final” sequences. Our method “PPR-Flow” (new name “PH-Flow” now) ranks 7th among all evaluated methods.

Figure 5: Method evaluation on the *test* set of Sintel benchmark with average end-point error (captured on 21-Nov-2014). We show 20 leading methods for the “Clean” and “Final” passes.